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Precalculus Final Exam Review

- 1) What are the **approximate** rectangular coordinates for the point with polar coordinates $(4, 60^\circ)$?

- A) (3.46, 2)
 B) (-3.81, -1.22)
 C) (2, 3.46)
 D) (-1.22, -3.81)

$$4 \cos 60 = 2$$

$$4 \sin 60 = 3.46$$

- 2) A sequence is shown below.

$5, 7, 11, 17, 25, 35, \dots$

Which is the recursive formula for this sequence?

- A) $t_n = n + 3(t_{n-1} + 2)$
 B) $t_n = (t_{n-1} + 1)(n - 4)$
 C) $t_n = 2(t_{n-1} + 2) - (n + 1)$
 D) $t_n = (t_{n-1} + 2)(n - 1)$

$$\begin{array}{cc} 5 + 2(2-1) & 7 + 2(3-1) \\ \downarrow & \downarrow \\ 7 & 11 \end{array}$$

- 3) A quadratic function, f , has a positive zero P and a minimum Q , such that the distance between P and Q is $\frac{5\sqrt{29}}{4}$. What is the equation for the quadratic function?

- A) $f(x) = -4x^2 + 2x - 3$ max
 B) $f(x) = x^2 + 3x - 4$ (1,0) (-1.5, -6.25)
 C) $f(x) = 2x^2 + x - 4$
 D) $f(x) = x^2 + 3x - 3$

- 4) Janet is investing money into an account that earns continuously compounded interest. Determine the amount of time it will take Janet's principal amount of \$4000 to double with a 5.5% interest rate.

- A) 10 years
 B) 12.6 years
 C) 14.3 years
 D) 15.1 years

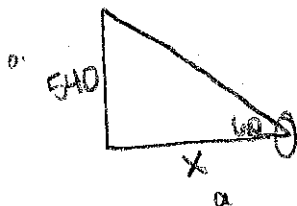
$$8000 = 4000 e^{.055t}$$

$$\frac{\ln 2}{.055} = t$$

$$t = 12.6$$

- 5) A person is standing on top of a building that is 540 feet tall. They are looking down at a statue at an angle of depression of 60° . **Approximately** how far away from the base of the building is the statue?

- A) 3458 feet
 B) 312 feet
 C) 84 feet
 D) 935 feet



$$\tan 60 = \frac{540}{x}$$

$$x = \frac{540}{\tan 60}$$

$$x = 311.77$$

6) Two functions are shown below.

$$f(x) = 2x^2$$

$$g(x) = -3x - 1$$

$$g(2) = -7$$

$$f(-1) = 2$$

What is the value of $f(g(2)) + g(f(-1))$

- A) 91
- B) 56
- C) 7
- D) 14

$$f(-7) + g(2)$$

$$98 + -7$$

7) A piecewise function is shown below. For what value of k does $\lim_{x \rightarrow -2} f(x)$ exist?

$$f(x) = \begin{cases} x^2 - k & x \leq -2 \\ x + 1 & x > -2 \end{cases}$$

$$x^2 - k = x + 1$$

$$(-2)^2 - k = -2 + 1$$

$$4 - k = -1$$

$$5 = k$$

- A) 4
- B) 2
- C) 5
- D) -5

8) What are the polar coordinates of $(2, 8)$ in radians?

- A) $(8.246, 1.326)$
- B) $(7.746, 1.158)$
- C) $(8.246, 0.245)$
- D) $(-0.291, 1.979)$

$$r^2 = 2^2 + 8^2$$

$$r = 8.246$$

$$\theta = \tan^{-1}(8/2)$$

$$\theta = 1.32$$

9) A sequence is shown below.

$$2, 6, 10, 14, 18, \dots$$

How many terms of the sequence must be added together for the sum to equal 1,152?

- A) 21
- B) 22
- C) 23
- D) 24

$$\sum_{k=1}^n 2 + (k-1)4 = 1152$$

guess & check

10) The first term of an infinite arithmetic sequence is -6 . The ninth term of the sequence is 68. What is the common difference of the sequence?

- A) 8.2 59.6
- B) 12 90
- C) 9.25 68
- D) 13.5 102

$$a_1 = -6$$

$$a_9 = 68$$

11) Which is true of the series shown below?

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$$

- A) The series diverges.
- B) The series converges to 1
- C) The series converges to $1/3$
- D) The series converges to $1/2$

$$r = -\frac{1}{2}$$

$$S = \frac{\frac{1}{2}}{1 - (-\frac{1}{2})} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

12) Kris recursively generated a sequence of five positive integers by starting with a positive integer, a_1 , and then applying the recursive formula $a_n = a_{n-1} + 5n$ to generate a_n for $n = 2, 3, 4$, and 5. If the value of a_4 was 202, what was the value of Karen's starting term, a_1 ?

- A) 167
- B) 176
- C) 125
- D) 157

$$\begin{aligned} 202 &= a_3 + 5(4) \\ 102 &= a_2 + 5(3) \\ 67 &= a_1 + 5(2) \\ 157 &= a_1 \end{aligned}$$

13) What is the distance between y -intercepts of the graph of $x = \frac{y^2 - 5}{?}$?

- A) 3.041
- B) 2.236
- C) 4.472
- D) 6.902

$$(2.24, 0) \quad (-2.24, 0^2)$$

$$4.48$$

14) Which is a solution set to $\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{x^2 - 6x + 8}$?

- A) $\{-1\}$
- B) $\{4\}$
- C) $\{-1, 4\}$
- D) No solution.

$$\begin{aligned} \frac{x(x-4) + (x-2)}{x^2 - 6x + 8} &= \frac{2}{x^2 - 6x + 8} \\ x^2 - 4x + x - 2 &= 2 \\ x^2 - 3x - 4 &= 0 \end{aligned}$$

15) What is the range of $y = 4\sin x$?

- A) $-1 \leq y \leq 1$
- B) $0 \leq y \leq 1$
- C) $-4 \leq y \leq 4$
- D) $-\infty \leq y \leq \infty$

↑
amp

$$(x - 4)(x + 1) = 0$$

~~$x = 4$~~ $x = -1$
undef

16) Johnny is standing 15 feet away from George.

- A plane is located in the sky at some point between where Johnny and George are standing.
- Johnny is looking up at the ^{plane} at an angle of elevation of 63°
- George is looking up at the ^{plane} at an angle of elevation of 78°

Approximately how far is the bird from Johnny?

- A) 0.98 ft
 B) 8 ft radian
 C) 23.3 ft degree
 D) 24.5 ft



$$\frac{\sin 39}{15} = \frac{\sin 78}{x}$$

$$x \sin 39 = 15 \sin 78$$

$$x = 23.31$$

17) What is the inverse function of $f(x) = 4x^2 - 16$

- A) $y = \frac{1}{4x^2 - 16}$
 B) $y^2 = x + 16$
 C) $y = \sqrt{0.25x + 4}$
 D) $y = \pm \sqrt{\frac{1}{4}x + 4}$

$$x = 4y^2 - 16$$

$$x + 16 = 4y^2$$

$$\pm \sqrt{\frac{x}{4} + 4} = y$$

18) What is the value of the limit shown below?

- A) $-\infty$
 B) 1
 C) 2
 D) Does Not Exist

$$\lim_{x \rightarrow -\infty} \frac{x+2}{x}$$

$$\frac{+}{+}$$

BOBO
 BOTN
 EATS DC

19) How many petals are on the polar rose which is represented by $r = 2\sin 4t$

- A) 2
 B) 4
 C) 6
 D) 8

θ in calc
 not 7

20) Convert the parametric equation below to rectangular form.

$$x = 2t + 4$$

$$y = t^2 - 1$$

$$\frac{x-4}{2} = t$$

- A) $f(x) = \frac{1}{4}x^2 - 2x + 3$
 B) $f(x) = 4x^2 + 2x - 4$
 C) $f(x) = \frac{1}{2}x^2 - 2x + 3$
 D) $f(x) = \frac{1}{2}x^2 + 2x - 1$

$$y = \left(\frac{x-4}{2}\right)^2 - 1$$

$$y = \frac{x^2 - 8x + 16}{4} - 1$$

$$y = \frac{1}{4}x^2 - 2x + 3$$

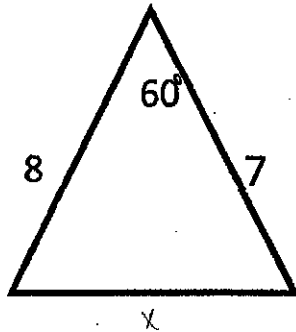
21) Which expression is equivalent to $\tan x \sin x + \cos x$?

- A) $\sin x \cos x$
- B) $\sec x$
- C) $\sin^2 x$
- D) $\frac{1}{\tan x}$

$$\frac{\sin x}{\cos x} \cdot \sin x + \cos x$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$$

22) Suppose that for each inch of ribbon needed to line the bottom of a party hat, the cost is \$0.21 per inch. Round to the nearest tenth of an inch. The diagram below shows the party hat. About how much will the ribbon cost?



$$x^2 = 7^2 + 8^2 - 2(7)(8)\cos 60$$

$$x = 7.5498$$

$$\frac{7.5498 \cdot 0.21}{\$1.58}$$

- A) \$7.50
- B) \$1.58
- C) \$219.7
- D) \$46.14

23) The function $P(x) = 400x - 50$ represent the profit for a company after selling x items. What is the inverse function of $P(x)$?

- A) $P^{-1}(x) = x + 50$
- B) $P^{-1}(x) = \frac{x+50}{400}$
- C) $P^{-1}(x) = -400x + 50$
- D) $P^{-1}(x) = \frac{x-400}{50}$

$$x = 400y - 50$$

$$\frac{x+50}{400}$$

24) Determine the type of conic section represented by the equation below and state the center.

$$\frac{(x-2)^2}{4} - \frac{(y+4)^2}{25}$$

- ~~A) Ellipse; (-2, 4)~~
- ~~B) Ellipse; (2, -4)~~
- C) Hyperbola; (-2, 4)
- D) Hyperbola; (2, -4)

25) A taxi driver charges a \$4 pick-up fee plus a tiered rate per mile. The table below displays the taxi driver's mileage rates.

| Miles | Rate Per Mile |
|---------------------|---------------|
| Miles 0 through 25 | \$0.25 |
| Miles 26 through 35 | \$0.20 |
| Miles 36 through 50 | \$0.15 |
| Miles 51 and up | \$0.10 |

What would the cost be for a taxi ride that was 38 miles?

$$4 + .25(25) + .20(10) + .15(3)$$

- A. \$12.70
 B. \$8.40
 C. \$6.50
 D. \$4.60

26) Revenue (in thousands) from sales of Compaq computers (a brand now extinct) are shown in the following table where t is the number of years since 1990.

| | | | | |
|--------|----|------|-----|-----|
| t | 0 | 1 | 5 | 10 |
| $R(t)$ | 50 | 33.5 | 5.2 | 0.7 |

$$b = .65$$

$$1 - r = .65$$

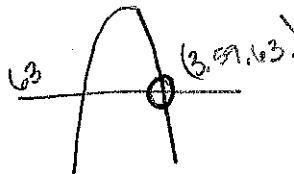
$$r = .35$$

Which type of function best models the data and why?

- A. A linear function, because the revenue is decreasing by 65% each year.
 B. A linear function, because the revenue is decreasing by 35% each year.
 C. An exponential function, because the revenue is decreasing by 65% each year.
 D. An exponential function, because the revenue is decreasing by 35% each year.

27. A tennis ball is launched straight upward from ground level with an initial velocity of 75 feet per second. The height, h (in feet above ground level), of the tennis ball t seconds after the launch is given by the function $h(t) = -16t^2 + 75t$. At approximately what value of t will the object have a height of 63 feet and be traveling downward?

- A. 2.34 seconds
 B. 1.10 seconds
 C. 3.59 seconds
 D. 0.73 seconds



28. What is the range of the function $f(x) = 3 - |2x + 4|$?

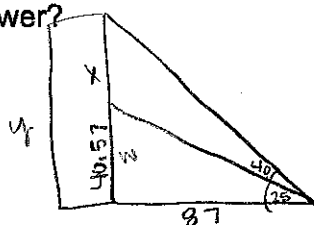
- A. $(-\infty, \infty)$
 B. $(-\infty, 3]$
 C. $[3, \infty)$
 D. $(-\infty, 4]$

29. What transformations have occurred to create the function $f(x) = 2\sqrt{3x-6}$ from the function $g(x) = \sqrt{x}$?

- A. The graph of the function has been stretched vertically, stretched horizontally, and shifted right 6.
- B. The graph of the function has been stretched vertically, shrunk horizontally, and shifted right 6.
- C. The graph of the function has been stretched vertically, stretched horizontally, and shifted right 2.
- D. The graph of the function has been stretched vertically, shrunk horizontally, and shifted right 2.

30. A radio station tower was built in two sections. From a point 87 feet from the base of the tower, the angle of elevation of the top of the first section is 25° , and the angle of elevation of the top of the second section is 40° . To the nearest foot, what is the height of the top section of the tower?

- A. 32 feet
- B. 86 feet
- C. 114 feet
- D. 41 feet



$$\tan 25 = \frac{w}{87}$$

$$w = 40.569$$

$$\tan 40 = \frac{y}{87}$$

$$y = 73$$

$$x = y - w$$

$$x = 32.4$$

31. Consider the two trigonometric functions:

$$f(x) = 4\cos\left(\frac{1}{2}x\right)$$

$$g(x) = -4\cos\left(\frac{1}{2}x + \frac{\pi}{3}\right)$$

How should the graph of $f(x)$ be changed to produce the graph of $g(x)$?

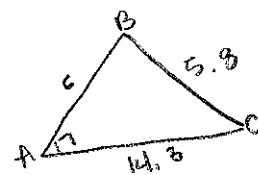
- A. Reflected over the y-axis and shifted left $\frac{\pi}{3}$
- B. Reflected over the x-axis and shifted left $\frac{\pi}{3}$
- C. Reflected over x-axis and shifted left $\frac{\pi}{6}$
- D. Reflected over x-axis and shifted left $\frac{2\pi}{3}$

* 32. Triangle ABC has the following properties:

- The angle at vertex A is 17° and the angle at vertex B is obtuse
- Side a is 5.8 units in length
- Side b is 14.3 units in length

What is the approximate length of the side opposite vertex C?

- A. Not possible
- B. 17.7 units
- C. 9.6 units
- D. 15.7 units



$$\frac{\sin 17}{5.8} = \frac{\sin B}{14.3}$$

$$B = 46.12$$

33. What value of x satisfies the equation $2\log_5(3x-1) = 6$.

- A. 42
- B. 209
- C. 2592
- D. 81

$$\begin{aligned} 5 \log_5(3x-1) &= 5 \cdot 3 \\ 3x-1 &= 125 \\ 3x &= 126 \\ x &= 42 \end{aligned}$$

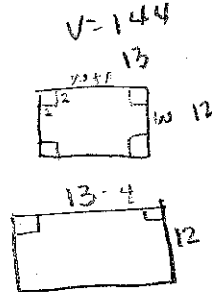
34. What is the inverse function of $f(x) = \log_7\left(\frac{x+3}{2}\right)$?

- A. $f^{-1}(x) = 7^x - 3$
- B. $f^{-1}(x) = \frac{7^x - 3}{2}$
- C. $f^{-1}(x) = 2(7^x) - 3$
- D. $f^{-1}(x) = 2(7^x) + 3$

$$\begin{aligned} x &= \log_7 \frac{y+3}{2} \\ 7^x &= \frac{y+3}{2} \\ 2(7^x) &= y+3 \\ 2(7^x) - 3 &= y \end{aligned}$$

35. You need to make a pizza box. You know that the box needs to be two inches deep, and needs to have a volume of 144 cubic inches. You will be forming the box by cutting out a large rectangle, and then cutting out the two-inch squares from the corners that will allow you to fold up the edges to make a two-inch-deep box. The rectangle needs to have a length that is 1 more than its width. What should be the length of the large rectangle?

- A. 12 inches
- B. 13 inches
- C. 8 inches
- D. 9 inches



$$\begin{aligned} V &= 144 \\ (w+1-4)(w-4) &= 144 \\ 2(w-3)(w-4) &= 144 \\ w^2 - 7w + 12 &= 72 \\ w^2 - 7w - 60 &= 0 \\ (w-12)(w+5) &= 0 \\ w &= 12 \quad w = -5 \end{aligned}$$

*36. A plane flies due east at a rate of 150 mph. The wind is blowing south at a rate of 50 mph. What is the resultant velocity?

- A. 158 mph @ 72° S of E
- B. 158 mph @ 72° E of S
- C. 158 mph @ 18° E of S
- D. 158 mph @ 18° S of E

37. What are the polar coordinates of the point $\left(\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$ where $0 \leq \theta \leq 360^\circ$?

- A. $(5, 300^\circ)$ and $(-5, 240^\circ)$
- B. $(5, 300^\circ)$ and $(-5, -240^\circ)$
- C. $(5, 300^\circ)$ and $(-5, -120^\circ)$
- D. $(5, 60^\circ)$ and $(5, -300^\circ)$

$$r = 5 \quad \theta = -60^\circ$$

38. Which equation is the rectangular form of the polar equation $r = \frac{4}{1 - \cos\theta}$?

- A. $x^2 + 8y = 16$
- B. $x^2 + y^2 = 16$
- C. $y^2 = 16$
- D. $y^2 - 8x = 16$

$$r = \frac{4}{1 - \cos \theta}$$

$$r - r \cos \theta = 4$$

$$r^2 = (4 + x)^2$$

$$x^2 + y^2 = 16 + 8x + x^2$$

$$y^2 = 8x + 16$$

39. What type of conic section is represented by $r = \frac{9}{6 + 10 \sin \theta}$?

- A. Circle
- B. Hyperbola
- C. Ellipse
- D. Parabola

$$6r + 10r \sin \theta = 9$$

$$6r = 9 - 10y$$

$$36(x^2 + y^2) = (9 - 10y)^2$$

40. When $a_1 = 13,000$, what is the sum of the infinite sequence defined by the equation

$$a_{n+1} = 0.75a_n$$

- A. 52000
- B. 17333
- C. 48000
- D. 51000

$$\frac{13000}{1 - 0.75}$$

41. The first term of an infinite geometric sequence is 9. The sum of the sequence is 45. What is the common ratio of the sequence?

- A. $\frac{1}{5}$
- B. $\frac{2}{3}$
- C. $\frac{4}{5}$
- D. 5

$$\frac{9}{1 - r} = 45$$

$$9 = 45 - 45r$$

$$-36 = -45r$$

$$r = \frac{4}{5}$$

42. What is the end behavior of the function $f(x) = \frac{50}{1 + 10(60)^x} + 4$?

- A. $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = \infty$
- B. $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = 54$
- C. $\lim_{x \rightarrow -\infty} f(x) = 4$ and $\lim_{x \rightarrow \infty} f(x) = \infty$
- D. $\lim_{x \rightarrow -\infty} f(x) = 4$ and $\lim_{x \rightarrow \infty} f(x) = 54$