9-3 Notes Geometric Sequences Name ______________________
Pre-Calculus

I. Evaluating Geometric Sequences

Another type of sequence is the geometric sequence. It occurs in applications to finance and population growth. In arithmetic sequences, we found that we added a number to the initial term to form the sequence. In geometric sequences, we start with a number and then generate a sequence by repeatedly multiplying a nonzero number r.

A **geometric sequence** may be defined recursively as $a_1 = a$, $\frac{a_n}{a_{n-1}} = r$, or as

$$a_1 = a, \qquad a_n = ra_{n-1}$$

where $a_1 = a$ and $r \neq 0$ are real numbers. The number a_1 is the first term, and the nonzero r is called the **common ratio**.

Identify the first term and the common ratio.

1. 2, 6, 18, 54, 162, …
2. $\{s_n\} = 2^{-n}$
3. $\{t_n\} = \{3 \cdot 4^n\}$

For a geometric sequence $\{a_n\}$ whose first term is a_1 and whose common ratio is r, the nth term is determined by the formula

$$a_n ,= a_1 r^{n-1} \quad r \neq 0$$

3. Given the sequence $10, 9, \frac{81}{10}, \frac{729}{100}, \ldots$
   a. Find the nth term.
   b. Find the 9^{th} term.
   c. Find a recursive formula for the sequence.

II. Finding the Sum of a Geometric Sequence

A. Partial Sum

Let $\{a_n\}$ be a geometric sequence with first term a_1 and common ratio r, where $r \neq 0$, $r \neq 1$. The sum S_n of the first n terms of $\{a_n\}$ is

$$S_n = a_1 + a_1 r + a_1 r^2 + \ldots + a_1 r^{n-1} = \sum_{k=1}^{n} a_1 r^{k-1} \quad \text{OR} \quad S_n = a_1 \cdot \frac{1-r^n}{1-r}$$

Find the sum S_n, for the first n terms of the geometric series:

$(\frac{1}{2})^n$

B. Infinite Series

If we think about a geometric series, we will realize that it will continue to go on and on. Hence it is called an **Infinite Geometric Series**. If we set about to find the sum of an infinite geometric series, it will do one of two things. First the sum S_n will approach a specific value or it **converges**. If not, then it is called a **divergent**.

If $|r| < 1$, then the infinite series $\sum_{k=1}^{n} a_1 r^{k-1}$ converges. Its sum is $\sum_{k=1}^{n} a_1 r^{k-1} = \frac{a_1}{1-r}$

Determine if the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{k=1}^{\infty} 2\left(\frac{2}{3}\right)^{k-1}$$

III. Applications

1. Show that the repeating decimal 0.999… = 11

2. Initially, a pendulum swings through an arc of 18 inches. On each successive swing, the length of the arc is 0.98 of the previous length.

   a. What is the length of the arc of the 10th swing?

   b. On which swing is the length of the arc first less than 12 inches?

   c. After 15 swings, what total distance will the pendulum have swung?

   d. When it stops, what total distance will the pendulum have swung?