9-1 Notes Sequences Name ______________________
Pre-Calculus

When we hear the word sequence we most likely think of a "sequence of events;" something that happens first, then second, and so on. Hey in math it is the same idea. Here a sequence deals with numerical outcomes that are first, second, and so on.
A **sequence** is a function f whose domain is the set of positive integers. The values f(1), f(2), f(3), … are called terms.

I. Evaluating a Sequence

1. Find the first six terms of the sequence.
$\{a_n\} = \left\{\frac{n-1}{n}\right\}$

2. Find the first six terms of the sequence.
$\{b_n\} = \left\{(-1)^{n+1}\left(\frac{2}{n}\right)\right\}$

3. Find the first six terms of the sequence.
$\{c_n\} = \begin{pmatrix} n \ \ if\ n\ is\ even \\ \frac{1}{n}\ if\ n\ is\ odd \end{pmatrix}$

II. Determining a sequence from a Pattern

Number the terms and see what happens between each term:

a) $e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \ldots$

b) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

c) 1, 3, 5, 7, …

d) 1, 4, 9, 16, 25, …

e) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$

III. Factorials

A factorial is a product of every integer from 1 to the number n.

$n! = n(n-1) \cdot 3 \cdot 2 \cdot 1$ for $n \geq 2$ where $0! = 1$ and $1! = 1$

Solve.

1. 9!

2. $\frac{12!}{10!}$

3. $\frac{3!\,7!}{4!}$

IV. A sequence Defined by a Recursive Formula

A second way of defining a sequence is to assign a value to the first term and specify the nth term by a formula or equation that involves on or more of the terms preceding it. The sequence is defined recursively, and the formula is a recursive formula.

Write the first 5 terms of the recursive sequence

$u_1 = 1,\ \ u_2 = 1,\ \ u_n = u_{n-2} + u_{n-1}$

V. Sigma Notation

Given a sequence $a1$, $a2$, $a3$, $a4$, . . . an. we can write the sum of the first n terms using summation notation, or sigma notation. The notation derives its name from the Greek Letter **Σ**. This corresponds to our S for "sum." The following notation is used:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots a_n$$

$\boldsymbol{k}$ is called the index of summation, it is the starting number for the sequence.

Write out each sum.

1. $\sum_{k=1}^{10} \frac{1}{k}$

2. $\sum_{k=1}^{n} k!$

Express each sum using summation notation

3. $1^2 + 2^2 + 3^2 + \ldots + 9^2$

4. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}}$

Properties.

If $\{a_n\}$ and $\{b_n\}$ are two sequences and c is a real number, then:

$$\sum_{k=1}^{n} (ca_k) = c\sum_{k=1}^{n} a_k \qquad \sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k \qquad \sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

$$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} a_k, \quad \text{where } 0 < j < n$$

Find the sums.

5. $\sum_{k=1}^{5} k^2$

6. $\sum_{j=3}^{5} \frac{1}{j}$

7. $\sum_{i=5}^{10} i$

8. $\sum_{i=1}^{6} 2$