8-4 Notes Real World Applications Name ______________________
Pre-Calculus

I. Compound Interest

Interest is the money paid for the use of money. Money borrowed is called **principal.** When you borrow money there is a rate of interest, expressed as a percent that is charged over the amount of time of the loan. Most often the loan is compounded a number of times per year.

Compound interest is calculated by the formula:

$$A\,(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

A(t)= amount after t years P=Principal r=interest rate per year
n=number of times compounded per year t=number of years

Calculate and compare the amount of money after one year using different compounding periods. How much money will you have after one year, if you invest $1000 at an annual rate of 10% compounded annually, semiannually, quarterly, monthly, and daily?

II. Continuously Compounded Interest

Continuously compounded interest uses the base e and is calculated by the formula:

$$A = Pe^{rt}$$

A(t)=amount after t years P=Principal r=interest rate per year t=number of years

1. Find the amount after 1 year if a principal investment of $1000 is invested at an interest rate of 10% per year, compounded continuously.

2. What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

III. Exponential Growth

Many natural phenomena have been found to follow the law that an amount N varies with time t according to the function. Here we have the Exponential Growth Model

$$N(t) = N_0e^{kt} \quad k > 0$$

N_0 = the original amount t = time k = constant that represent the growth rate

A colony of bacteria grows according to the law of uninhibited growth according to the function: $N(t)$ = 100e 0.045t Where N is measured in grams and t is measured in days.

a. Determine the initial amount of bacteria.

b. What is the growth rate of the bacteria?

c. Graph the function.

d. What is the population after 5 days?

e. How long will it take for the population to reach 140 grams?

f. What is the doubling time for the population?

IV. Radioactive Decay

The amount A of a radioactive material present at time t is given by:

$$A(t) = A_0e^{kt} \quad k > 0$$

A_0 = the original amount t = time k = a negative number that represent the rate of decay

Traces of burned wood along with ancient stone tools in an archeological dig in Chile were found to contain approximately 1.67% of the original amount of carbon 14. The half-life of carbon 14 is 5700 years.

a. Approximately when was the tree cut and burned?

b. Graph the relation between the percentage of carbon 14 remaining and time.

V. Newton's Law of Cooling

Newton's Law of Cooling stated that the temperature of a heated object decreases exponentially over time toward the temperature of the surrounding medium. The temperature u of a heated object at a given time t can be modeled by:

$$u(t) = T + (u_0 - T)e^{kt} \quad k > 0$$

u(t) = temperature T = surrounding temperature u_0 = initial temperature

k = constant (negative number) t = time

An object is heated to 100°and is then allowed to cool in a room whose air temperature is 30°C.

a. If the temperature of the object is 80°Cafter 5 minutes, when will its temperature be 50°C?

b. Graph the relation found between the temperature and time.

c. Determine the elapsed time before the object is 35°.