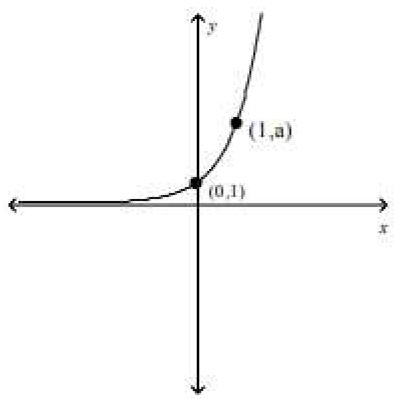
8-1 Notes Exponentials & Logarithms Name ______________________
Pre-Calculus

The exponential function is one of the most important functions in mathematics. The function is used to model the natural process of population growth and radioactive decay. It is also important in finances such interest and depreciation. The exponential function with base a is defined for all real numbers x by:

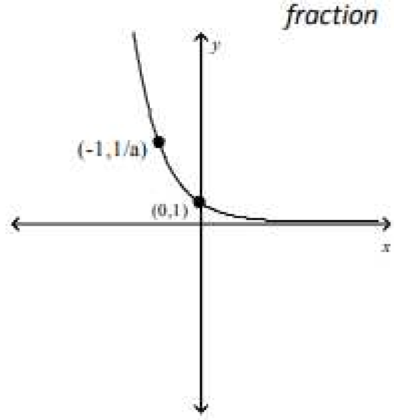
$$f(x) = Ca^x$$
$$y = Ca^x$$

where a > 0 and a ≠ 1
a is the growth factor C is the initial value

Growth:
$f(x) = a^x, \quad a \neq 0$



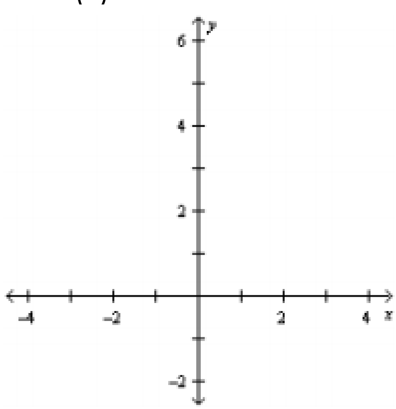
Decay:
$f(x) = a^x, \quad 0 < a < 1$
fraction



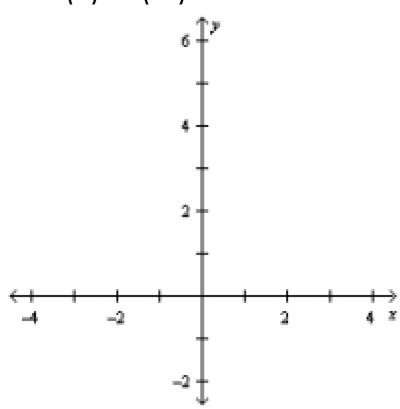
- pass through a common point of (0, 1)
- domain= (−∞, ∞)
- range (0, ∞)
- asymptote at y = 0

I. Graphing Exponentials

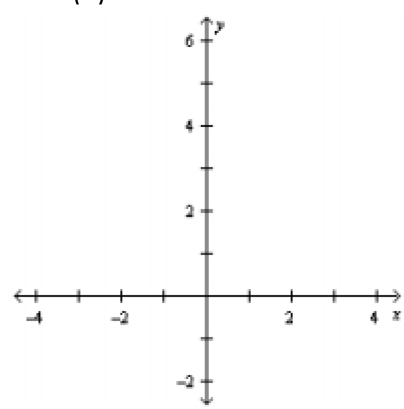
1. f(x) = 2^x



2. f(x) = $(½)^x$



3. f(x) = $2^{-x} – 3$



II. The Natural Exponential Number

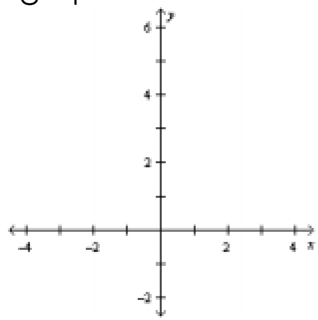
A chap named Leonard Euler named this irrational number e = 2.71828 …. Applications include the naturally occurring processes of continuous growth and decay and may also be used to model any growth/decay that is continuous.

The number e is defined as the number the number in the expression: $(\mathbf{1} + \mathbf{1}/\boldsymbol{n}\ )^{\boldsymbol{n}}$
The main thing we need to recognize that and exponential equation is expressed with a base value of e → y = e^x

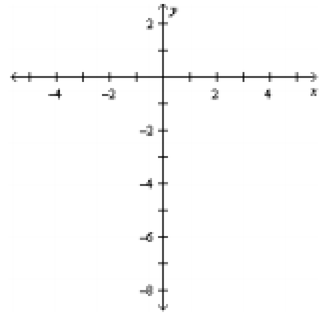
A. Graph using transformations

$f(x) = -e^{x-3}$

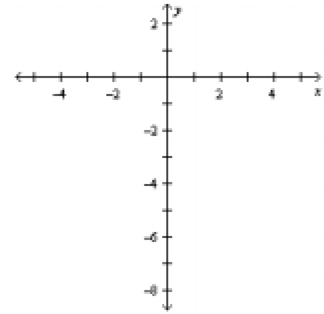
a. graph e^x



b. reflection



c. translation



B. Solve Exponential Equations

1. $4^{2x-1} = 8^x$

2. $e^{-x^2} = (e^x)^2 \frac{1}{e^3}$

III. Logarithmic Functions

The inverse of an exponential function is a logarithmic function.
Let a be a positive number with a not equal to 1.
The logarithmic function with base a is defined by:

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

$$\log_a x = y \quad \text{if and only if} \quad a^y = x$$

[translation: whatever you are taking the log of has to be greater than zero]
start with the base and move in a counterclockwise fashion

A. Change each logarithmic statement to an equivalent statement involving an exponent.

1. $\log_a 4 = 5$
2. $\log_e b = -3$
3. $\log_3 5 = c$

B. Change each exponential statement to an equivalent statement involving a logarithm.

4. $1.2^3 = m$
5. $e^b = 9$
6. $a^4 = 24$

C. Find the exact value.

7. $\log_2 16$
8. $\log_3 \frac{1}{27}$

D. Fid the domain of each logarithmic function.

9. $f(x) = \log_2(x + 3)$
10. $g(x) = \log_5 \left(\frac{1+x}{1-x}\right)$

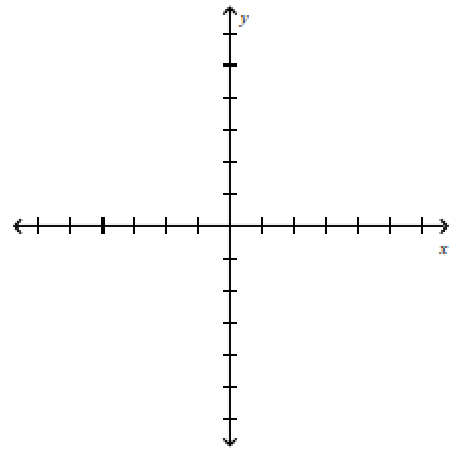
IV. Graphing Logarithmic Functions

Knowing the general form of the graph of the log function is a short cut for graphing.

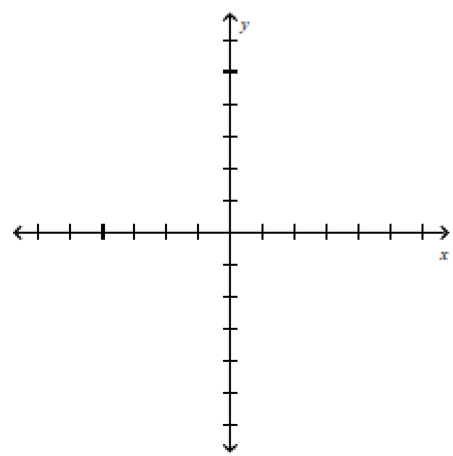
1. Write in its equivalent exponential form
2. Find the inverse; $x\ is\ y\ and\ y\ is\ x$, solve for y
3. Graph the log function's inverse, and reflect the exponential graph across the line of symmetry $y = x$.

Graph, determine the domain, range and vertical asymptote.

1. $y = \log_2 x$



2. $y = \log_{1/3} x$



V. The Natural and Common Logarithm

The Natural Logarithm is a logarithm with the base e. It is written with the abbreviation of ln.

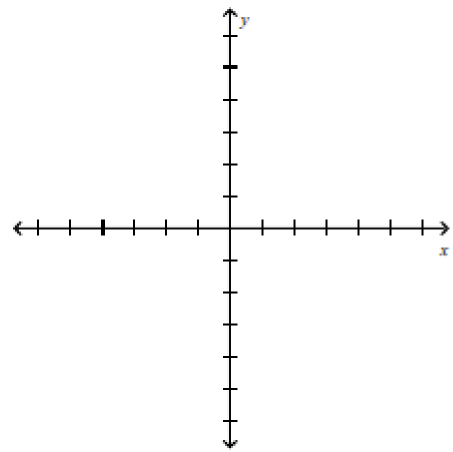
$y = \ln x$, if and only if $x = e^y$

$y = \log_x$, if and only if $x = 10^y$

The Common Logarithm is a logarithm with the base 10. It is written without the base number.

A. Graph, determine the domain, range and vertical asymptote. Identify the inverse and the domain and range of the inverse.

$f(x) = - \ln(x - 2)$



B. Solving Logarithmic Equations

1. $\log_3(4x - 7) = 2$

2. $\log_x 64 = 2$

C. Using Logarithms to Solve and Exponential Equation

3. $e^{2x} = 5$