6-4 Notes The Dot Product Name _____
Pre-Calculus

I. Dot Product

If $v = a_1i + b_1j = \langle a_1, b_1 \rangle$ and $w = a_2i + b_2j = \langle a_2, b_2 \rangle$ are vectors, then their dot product, denoted by $\boldsymbol{v} \cdot \boldsymbol{w}$, is defined by $\boldsymbol{v \cdot w = a_1a_2 + b_1b_2}$

Given $v = 2i - 3j$ and $w = 5i + 3j$, find the following dot products.

1. $v \cdot w$

2. $w \cdot v$

3. $v \cdot v$

4. $w \cdot w$

5. $\|v\|$

6. $\|w\|$

The following properties of the Dot Product are useful in solving problems involving the Dot Product:

$u \cdot v = v \cdot u$ $\quad v \cdot v = \|v\|^2$ $\quad 0 \cdot v = 0$

$(au) \cdot v = a(u \cdot v) = u \cdot (av)$ $\quad u \cdot (v + w) = u \cdot v + u \cdot w$

II. The Dot Product Theorem

If we have u and v be vectors with initial points at the origin, the angle θ that is between u and v is $0 < \theta < \pi$.

Find the angle θ between $u = 4i - 3j$ and $v = 2i + 5j$.

$$\boldsymbol{u \cdot v = \|u\|\|v\|cos\theta}$$

$$\boldsymbol{cos\theta = \frac{u \cdot v}{\|u\|\|v\|}}$$

III. Parallel & Perpendicular

A. Orthogonal Vectors (a.k.a. perpendicular)

Two vectors v and w are orthogonal, a.k.a. perpendicular, if and only if $v \cdot w = 0$

1. Determine whether the vectors are perpendicular.

$v = 2i - j$ $w = 31 + 6j$

B. Parallel Vectors

Two vectors v and w are parallel if they are "multiples" of each other.

Determine whether the vectors in each pair are parallel.

2. $v = 2i - j$ and $w = 6i - 3j$

3. $w = 3i + 4j$ and $r = 5i + 2j$

IV. Work

Work equals force times distance – $W = F \cdot D$

English units of force is pounds (lbs).

When the force acting on the object is at an angle, remember to break it into its horizontal and vertical components.

1. A girl is pulling a wagon with a force of 50 pounds. How much work is done in moving the wagon 100 feet if the handle makes an angle of 30o with the ground?