6-2 Notes Graphing Polar Equations   Name ______________________
Pre-Calculus

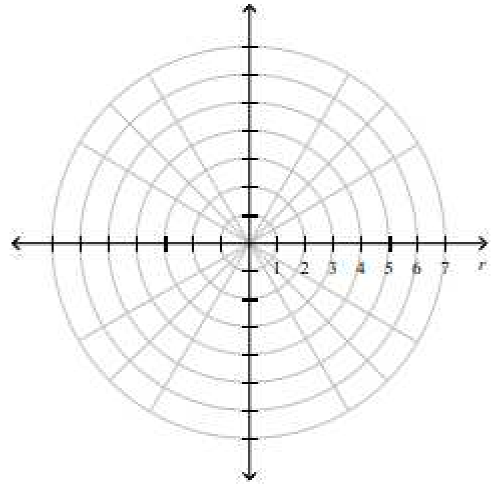
To plot points with polar coordinates it is convenient to use a polar grid. It is sort of like the unit circle superimposed with graph paper.

I. Special Graphs

θ = constant – graphs a line at angle θ
r = constant – graphs a circle of radius r

Sketch the graph of the equations and express the equations in rectangular coordinates:

$\theta = \pi/3$

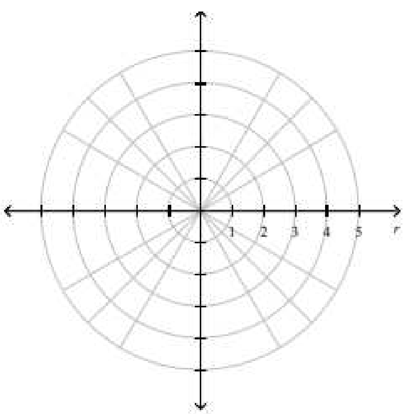


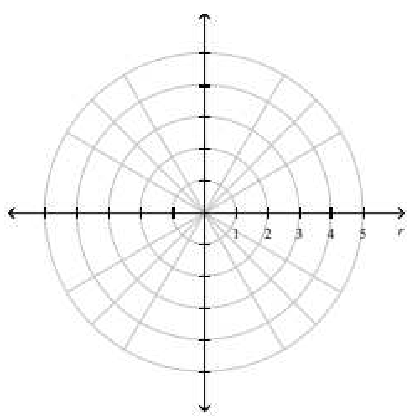
r = 3

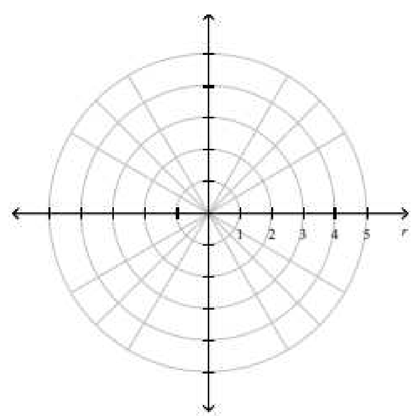
II. Graphing a Polar Equation of a Line

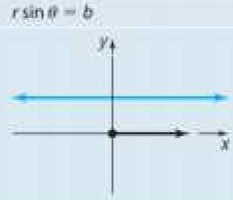
Some equations can easily be expressed in rectangular coordinates. If this is the case, then convert to rectangular coordinates.
Identify and graph the equation.

1. $\theta = \pi/4$ 2. $r \sin \theta = 2$ 3. $r \cos \theta = -3$







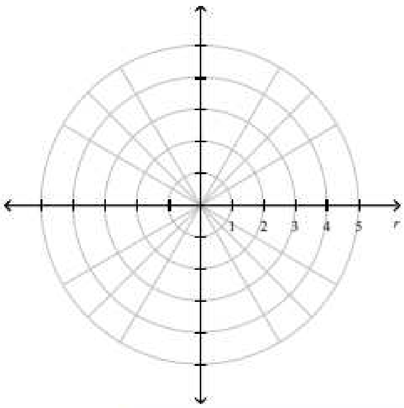
Lines			
Description	Line passing through the pole making an angle α with the polar axis	Vertical line	Horizontal line
Rectangular equation	$y = (\tan \alpha)x$	$x = a$	$y = b$
Polar equation	$\theta = \alpha$	$r \cos \theta = a$	$r \sin \theta = b$
Typical graph			

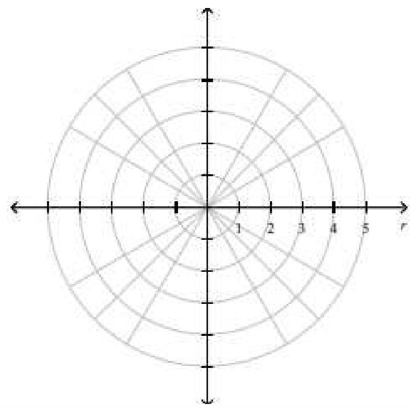
III. Graphing a Polar Equation of a Circle

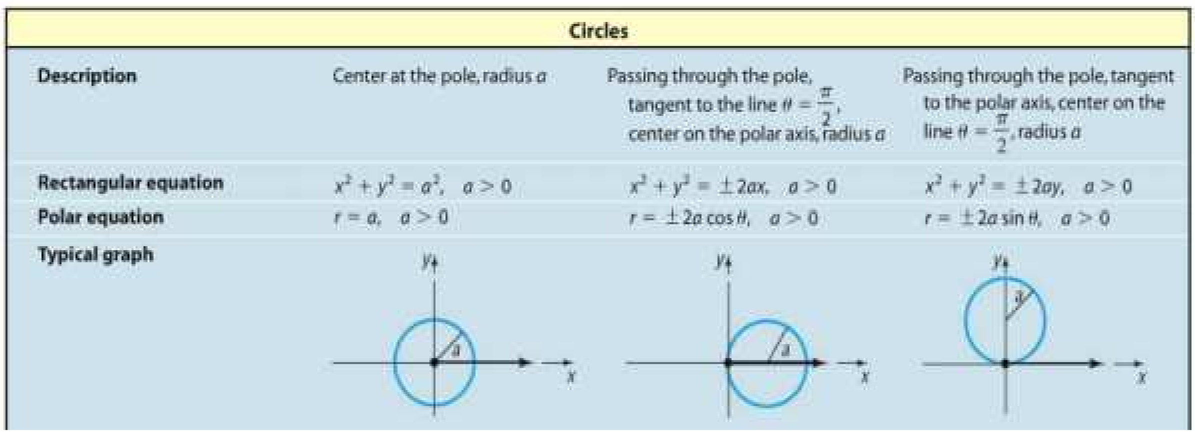
Sketch the polar equation (transform the equation into its rectangular form).

1. $r = 4 \sin \theta$

2. $r = -2 \cos \theta$

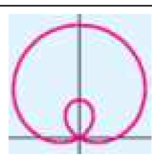
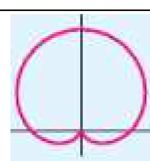
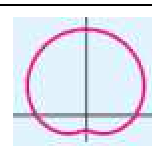






Circles			
Description	Center at the pole, radius a	Passing through the pole, tangent to the line $\theta = \frac{\pi}{2}$, center on the polar axis, radius a	Passing through the pole, tangent to the polar axis, center on the line $\theta = \frac{\pi}{2}$, radius a
Rectangular equation	$x^2 + y^2 = a^2, \quad a > 0$	$x^2 + y^2 = \pm 2ax, \quad a > 0$	$x^2 + y^2 = \pm 2ay, \quad a > 0$
Polar equation	$r = a, \quad a > 0$	$r = \pm 2a \cos \theta, \quad a > 0$	$r = \pm 2a \sin \theta, \quad a > 0$
Typical graph			

IV. Other Equations

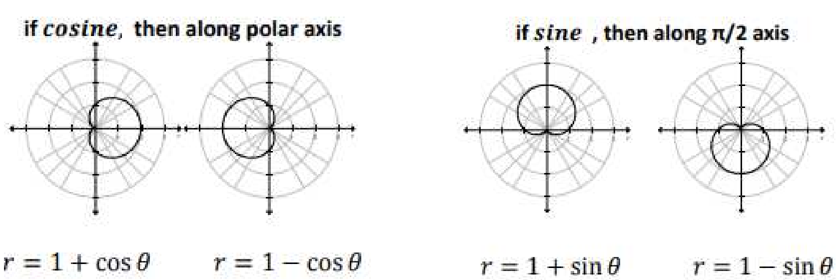
Name	Limaçon (inner loop)	Cardioid	Limaçon (dimple)
Polar Equation	$r = a \pm b \cos \theta$ $r = a \pm b \sin \theta$ $a < b$	$r = a \pm a \cos \theta$ $r = a \pm a \sin \theta$ $a = b$	$r = a \pm b \cos \theta$ $r = a \pm b \sin \theta$ $a > b$
Graph			

Equations in terms of cosine will be symmetrical about the polar axis (horizontal). Equations in terms of sine will be symmetrical about the π/2 axis (vertical).

A. Cardioids

$a = b$

distance on axis is 2a



1. Graph $r = 2 - 2 \sin\theta$.

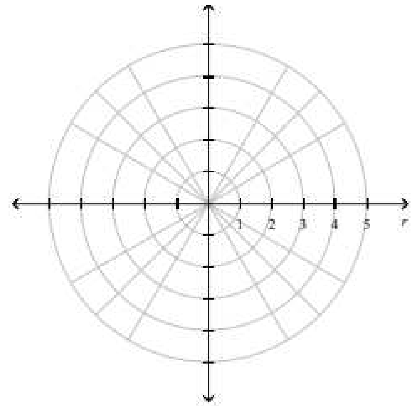
a = _____ b = _____

The number indicate a shape of ____________

equation has sine so along ________ axis

negative means __________________

length = ________

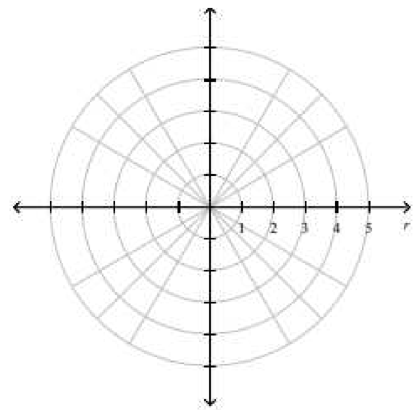


B. Limaçon Graphs

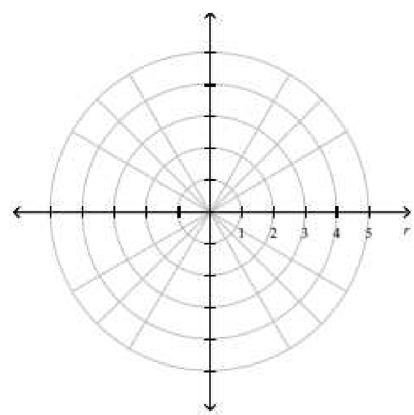
$r = a \pm b \cos \theta$ if cosine: along polar axis a > b no inner loop

$r = a \pm b \sin \theta$ if sine: along $\pi/2$ axis a < b inner loop

2. Graph $r = 3 + 2 \cos \theta$



3. Graph $r = 1 + 2 \cos \theta$



V. More Equations

A. Roses

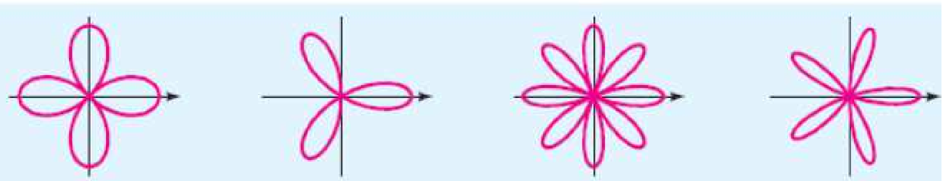
Roses

$r = a \sin n\theta$

$r = a \cos n\theta$

n-leaved if n is odd

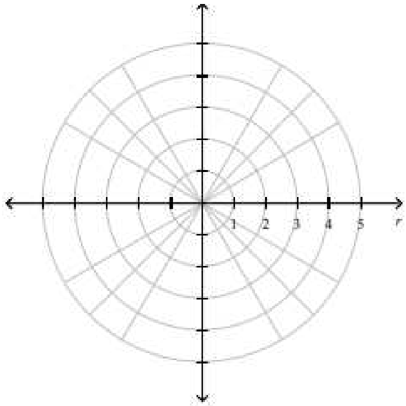
$2n$-leaved if n is even



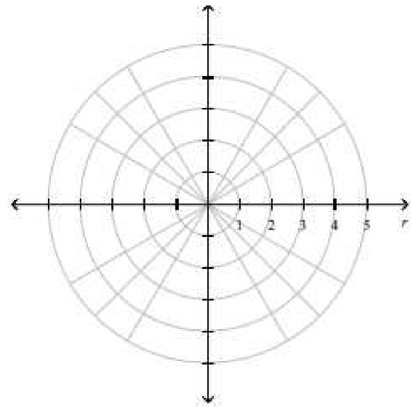
$r = a \cos 2\theta$ 4-leaved rose | $r = a \cos 3\theta$ 3-leaved rose | $r = a \cos 4\theta$ 8-leaved rose | $r = a \cos 5\theta$ 5-leaved rose

Graph n-leaved rose.

1. $r = 2 \sin 3\theta$

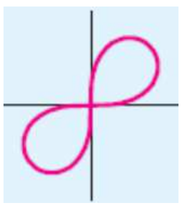


2. $r = 2 \cos 2\theta$

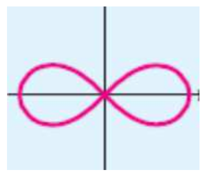


B. Lemniscates

$r^2 = a^2 \sin 2\theta$



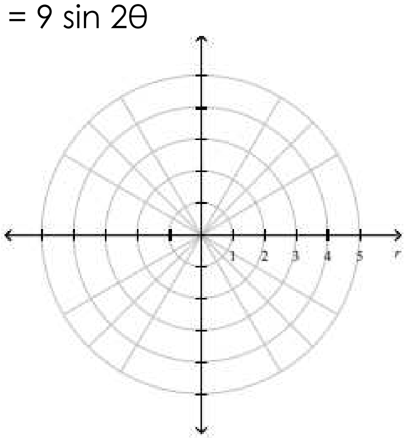
$r^2 = a^2 \cos 2\theta$



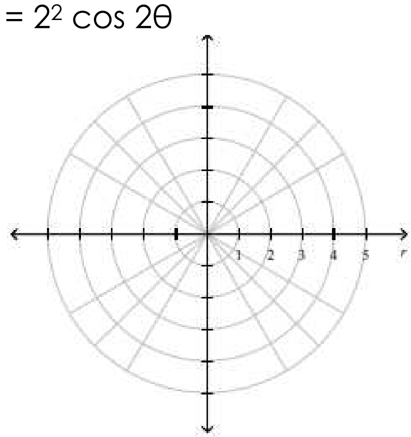
a = petal length

Graph figure-8 curve.

3. $r^2 = 9 \sin 2\theta$



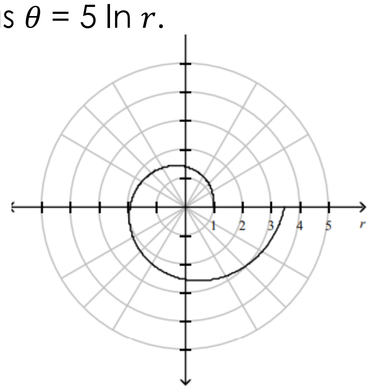
4. $r^2 = 2^2 \cos 2\theta$



C. Spirals

Graphing a Polar Equation (spiral) It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

There are several equations that will produce a spiral. The logarithmic spiral $r = e\ \theta/5$ may be written as $\theta = 5 \ln r$.



Archimedes Spiral is in the form of $r = a\theta$