5-2 Notes Law of Sines Name ________________________

Pre-Calculus

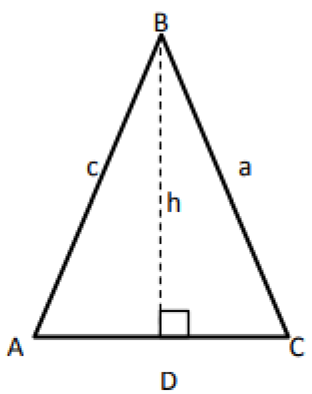
Until now, our work with triangles and trig functions has been limited to right triangles. However, what we have learned to date in this class allows us to work with all triangles. A triangle has three sides and three angles. If we know any three of these six measurements with one of the known measurements being a side, the other three measures may be found.

A triangle that does not contain a right angle is known as an **oblique triangle**. The data required to solve an oblique triangle may be listed in four cases. The first two cases are studied in this section and the other two will be studied in the next section.

Case 1 – One side and two angles are known (SAA or ASA).
Case 2 – Two sides and one angle not included between the two sides are known (SSA).
This case may lead to more than one triangle.
Case 3 – Two sides and the included angle are known (SAS).
Case 4 – Three sides are known (SSS).

I. Law of Sines

The Law of Sines may be derived either by using an acute or obtuse triangle.



While ΔABC is acute if we drop a perpendicular we get right triangles, $\Delta ABD\ and\ \Delta BCD$. Using our trig functions we find:

$$\sin A = \frac{h}{c} \quad or \quad h = c\sin A$$

$$\sin C = \frac{h}{a} \quad or \quad h = a\sin C$$

$$\rightarrow \quad a\sin C = c\sin A \quad or \quad \frac{a}{\sin A} = \frac{c}{\sin C}$$

Similarly, you may find a relationship for angle B and then you will have the final form of the Law of Sines.

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or using reciprocals

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Translation: according to the Law of Sines, the ratio of the length of a side of a triangle to the sine of the angle opposite the side is proportionate to the other ratios of sides to sine of angle opposite to the side.

Strategy: When using the Law of Sines, select an equation so that the unknown variable is in the numerator and all other variables are known. DRAW THE TRIANGLE AND CAREFULLY LABEL!!!!

A. SAA

Solve the triangle: A = 40˚, B = 60˚, a = 4

B. ASA

Solve the triangle: A = 35˚, B = 15˚, c = 5

II. The Ambiguous Case

Can there be more than one $\angle B$? This is what we have in Case 2, SSA. Case 2 is referred to as the **ambiguous case**. The information given may result in one triangle, two triangles or perhaps no triangle at all.

A. SSA – one solution

Solve the triangle: A = 40˚, a = 3, b = 2

B. SSA – two solutions

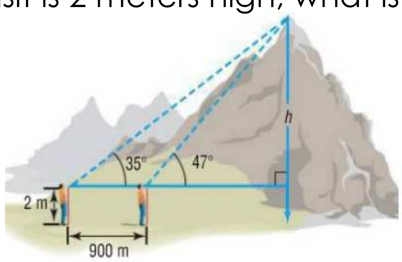
Solve the triangle: A = 35˚, a = 6, b = 8

C. SSA – no solution

Solve the triangle: C = 50˚, a = 2, c = 1

III. Applications

1. To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance of 900 meters apart on a direct line to the mountain. The first observation results in an angle of elevation of 47°, and the second results in an angle of elevation of 35°. If the transit is 2 meters high, what is the height, h, of the mountain?



2. Coast Guard Station Zulu is located 120 miles due west of Station X-ray. A ship at sea sends an SOS call that is received by each station. The call to Station Zulu indicates that the bearing of the ship from Zulu is $N40°E$. The call to Station X-ray indicates that the bearing of the ship from X-ray is $N30°W$. How far is each station from the ship? If a helicopter capable of flying 200 mph is dispatched from the nearer station to the ship how long will it take to reach the ship?