**4-1 Notes** Inverse Trig Functions Name______________________

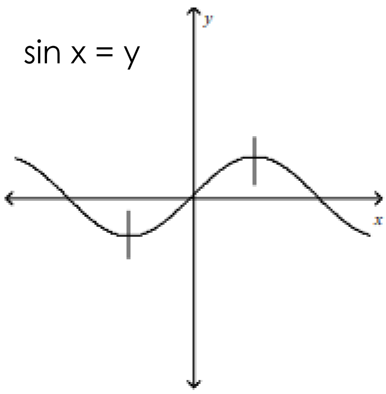
Pre-Calculus

**Recall:** If a function is one-to-one it has an inverse function (one function undoes the other). To make a function one-to-one, place restrictions on the domain. While trig functions are of course functions, they are not all one-to-one. By placing limits on the domain, a trig function may be forced into being one-to-one. Remember to find the inverse simply switch the $x$ and $y$ values; $x \text{ is } y$ and $y \text{ is } x$. Switch the domain and range.

**I. Inverse Sine Function**

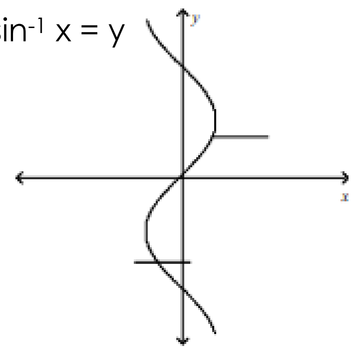
**sin$^{-1}$** also known as <u>arcsine</u> written as <u>arcsin</u>

sin x = y



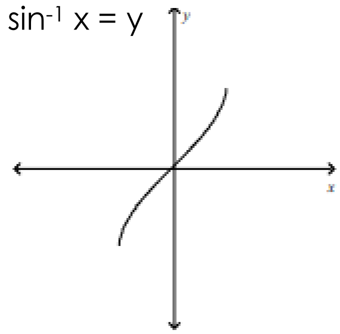
Domain: (∞, ∞)
(restricted D): [-$\pi$/2, $\pi$/2]
Range: [-1, 1]

sin$^{-1}$ x = y



Restrict Sine's domain to allow for an inverse function

sin$^{-1}$ x = y



Domain: [-1, 1]
Range: [-$\pi$/2, $\pi$/2]

A. Finding the exact value of an inverse sine function
we are looking for an angle $\theta$, where $-\pi/2 \le \theta \le \pi/2$

1. sin$^{-1}$ (1)
2. sin$^{-1}$ (-1/2)
3. sin$^{-1}$ (3/2)

B. Approximate values of inverse sine functions may be found using a calculator. Express the answer in radians rounded to 2 decimal places.

4. sin$^{-1}$ (1/3)
5. sin$^{-1}$ (-1/4)

C. In the terms of the sine function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

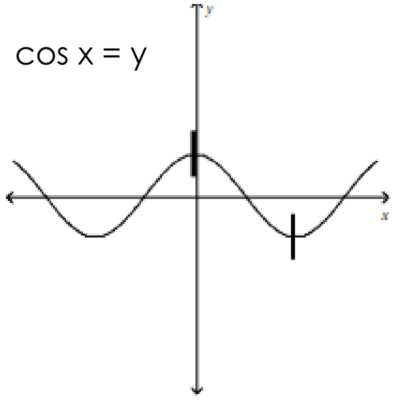
$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

Find the exact value of composite functions.

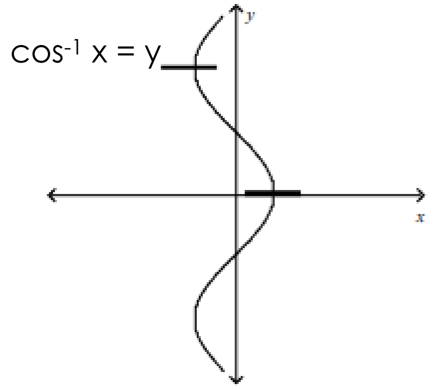
6. sin$^{-1}$ (sin π/4)
7. sin$^{-1}$ (sin 2π/3)

## II. Inverse Cosine Function

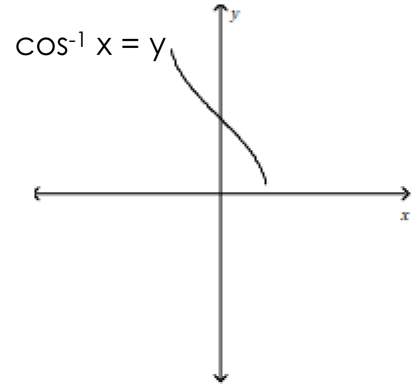
**cos$^{-1}$** also known as <u>arccosine</u> written as <u>arcos</u>




Domain: (∞, ∞)
(restricted D): [0, $\pi$]
Range: [-1, 1]




Restrict Cosine's domain to allow for an inverse function




Domain: [-1, 1]
Range: [0, $\pi$]

A. Finding the exact value of an inverse cosine function
we are looking for an angle $\theta$, where $0 \leq \theta \leq \pi$

1. $\cos^{-1} 0$

2. $\cos^{-1} (-\sqrt{2}/2)$

B. In the terms of the cosine function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \quad \text{where } 0 \leq x \leq \pi$$

$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \quad \text{where } -1 \leq x \leq 1$$

Find the exact value of the composite functions.

3. $\cos^{-1}(\cos(\pi/12))$

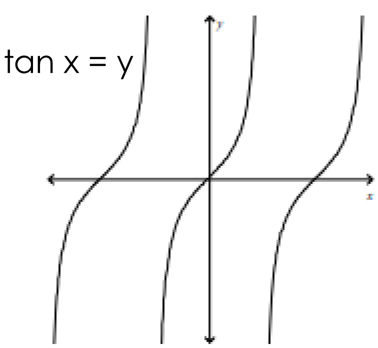
4. $\cos^{-1} [\cos (-2\pi/3)]$

5. $\cos (\cos^{-1} \pi)$

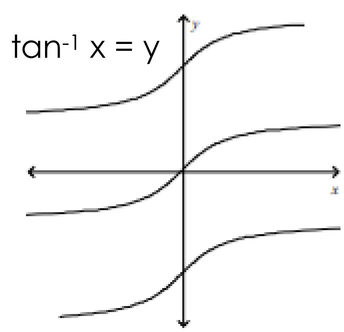
6. $\cos (\cos^{-1} (\sqrt{3}/2))$

## III. Inverse Tangent Function

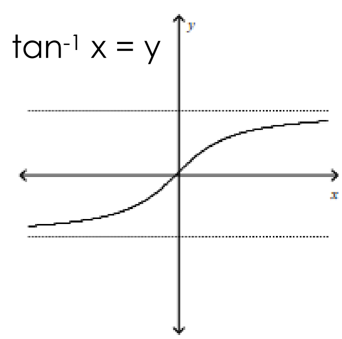
**tan$^{-1}$** also known as <u>arctangent</u> written as <u>arctan</u>




Domain: (∞, ∞)
≠ odd multiples
(restricted D): [-$\pi$/2, $\pi$/2]
Range: [-∞, ∞]




Restrict Tangent's domain to allow for an inverse function




Domain: [-∞, ∞]
Range: [-$\pi$/2, $\pi$/2]

A. Evaluate the inverse tangent functions; find $\theta$ $for$ $-\pi/2 \leq \theta \leq \pi/2$

1. $\tan^{-1} 1$
2. $\tan^{-1} -\sqrt{3}$
3. $\tan^{-1} -20$

B. In the terms of the tangent function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where } -\infty < x < \infty$$

**IV. Evaluate a trig function involving inverse trig functions.**

1. Find the exact value of $\sin(\tan^{-1}(½))$

- let θ equal the inverse function
- by definition: $\theta = \tan^{-1}(½)$ so $\tan\theta = ½$
- set up a triangle in which $\tan\theta = ½$

2. $\cos[\sin^{-1}(-1/3)]$

3. $\tan[\cos^{-1}(-1/3)]$

4. $\cos^{-1}[\tan(-\pi/4)]$

5. Write a trig expression as an algebraic expression $\sin(\tan^{-1} u)$