2-6 Notes Properties of Rational Functions Name ______________________
Pre-Calculus

I. Rational Functions

When dealing with ratios of integers, they are identified as rational numbers. When we look at ratios of polynomials, we call them rational functions.

A **rational function** is a function in the form $R(x) = \frac{p(x)}{q(x)}$ where p and q are polynomial functions and q ≠ 0. The domain of a rational function is the set of all real numbers except those for which the denominator q is 0.

A. Find the domain of the rational function.

1. $R(x) = \frac{2x^3 - 4}{x + 5}$

2. $R(x) = \frac{1}{x^2 - 4}$

3. $R(x) = \frac{x^3}{x^2 + 1}$

4. $R(x) = \frac{x^2 - 1}{x - 1}$

B. Graph and analyze. What happens at x = 0? As x → 0? As x →∞?

5. $R(x) = \frac{1}{x}$

6. $H(x) = \frac{1}{x^2}$

C. Graph the rational function using transformations.

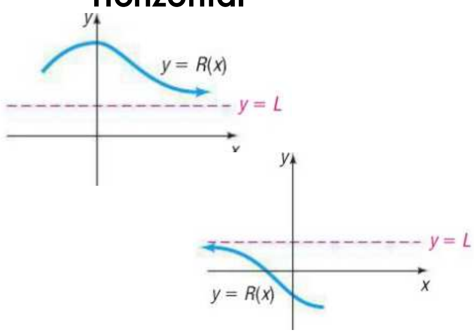
$R(x) = \frac{1}{(x-2)^2} + 1$

II. Asymptotes

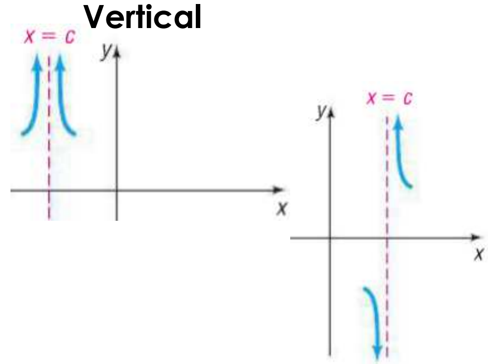
Let R denote a function: If, as x → -∞ or as x → ∞, the values of R(x) approach some fixed number L, then the line y =L is a **horizontal asymptote** of the graph of R. If as x approaches some number c, the values |R(x)| → ∞, then the line x = c is a **vertical asymptote** of the graph R.

A graph MAY cross a horizontal asymptote, but it will NEVER cross a vertical asymptote.

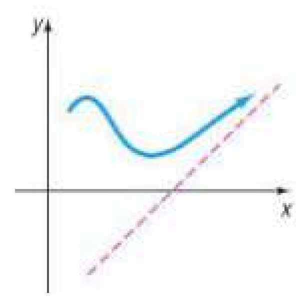
Horizontal



Vertical



Oblique



A. **Vertical Asymptotes** will occur at x-values that make the denominator = 0.

Find the VAs, if any, of the graph of each rational function.

1. $F(x) = \frac{5x^2}{3+x}$

2. $R(x) = \frac{x}{x^2 - 4}$

3. $H(x) = \frac{x^2}{x^2 + 1}$

4. $G(x) = \frac{x^2 - 9}{x^2 + 4x - 21}$

B. For **Horizontal & Oblique Asymptotes** we need to compare the degree of the numerator (t) and the degree of the denominator (b).

If t < b, HA at x-axis (y = 0).

If t > b, no HA but could have OA

* If t is exactly 1 degree larger than b, graph has OA. Find equation of OA by synthetic or long division, remainder doesn't matter.

If t = b, divide leading coefficients to find HA

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Find the horizontal asymptotes if any for the graph of:

5. $R(x) = \frac{x - 12}{4x^2 + x + 1}$

6. $H(x) = \frac{3x^4 - x^2}{x^3 - x^2 + 1}$

7. $R(x) = \frac{8x^2 - x + 2}{4x^2 - 1}$

8. $G(x) = \frac{2x^5 - x^3 + 2}{x^3 - 1}$