2.5 Complex Zeros and the Fundamental Theorem of Algebra

Fundamental Theorem of Algebra

A polynomial function of x with degree n has n ______________________________ (real and nonreal). Some of these zeros may be repeated.

Linear Factorization Theorem

If $f(x)$ is a polynomial function $n > 0$, then $f(x)$ has precisely ______________________.

Fundamental Polynomial Connections in the Complex Case

The following statements about a polynomial function ***f*** are equivalent, if ***k*** is a complex number:

1. ____
2. ____
3. ____

Complex Conjugate Zeros

Suppose that $f(x)$ is a polynomial function with real coefficients. If a and b are real numbers with $b \neq 0$, and _________________ is a zero of $f(x)$, then _________________ its _______________________________ is also a zero.

Fundamental Polynomial Connections

EX #1: Write the polynomial function in standard form, and identify the zeros of the function and the x-intercepts of its graph. $f(x) = (x+2)\left(x - i\sqrt{3}\right)\left(x + i\sqrt{3}\right)$

Finding a Polynomial Given Complex Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include the following:

EX#2: $\{-3, 4, (2-i)\}$

EX #3: $x = 1$ (multiplicity 2); $x = 1 - i$

Factoring a Polynomial with Complex Zeros

EX #4: Find all the zeros of $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$ and factor $f(x)$ over the set of complex numbers.

EX #5: The complex number $z = 2i$ is a zero of the polynomial. Find the remaining zeros of $f(x)$ and factor over complex numbers: $f(x) = 2x^4 + x^3 - 7x^2 + 4x - 60$

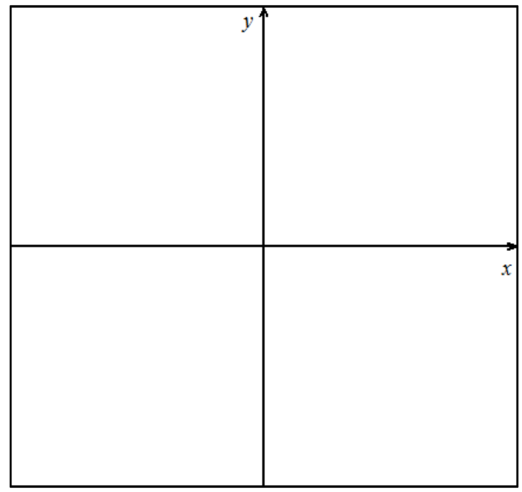
Factoring Over Real Numbers

EX #6: Write $f(x) = 3x^3 - 2x^2 + x - 2$ as a product of linear and irreducible quadratic factors, each with real coefficients. That is, factor over real numbers.

Sketching A Graph with Repeating Zeros

EX #7: Sketch the graph of the polynomial function with the given zeros, multiplicities, and sign of leading coefficient

A. –3 (multiplicity 2) ; 4 (multiplicity 3); $a > 0$



B. –1 (multiplicity 2); 3 (multiplicity 2) ; $a < 0$