**2-4 Notes** Real Zeros of Polynomials    Name ________________________
Pre-Calculus

## I. Dividing Polynomials

This is a very similar process to the old long division we did in elementary school.

$$\frac{38}{7} = 5 + \frac{3}{7} \text{ where } \frac{3}{7} \text{ is the fractional remainder}$$

Proper format for division of polynomials:

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad f(x) = q(x)g(x) + r(x)$$

(dividend: $f(x)$, quotient: $q(x)$, divisor: $g(x)$, remainder: $r(x)$)

A. Divide

1. $6x^2 - 26x + 12$ by $x - 4$
2. $8x^4 + 6x^2 - 3x + 1$ by $2x^2 - x + 2$

B. Synthetic Division

This is a quick method for dividing polynomials when the divisor is of the form $x - c$.

3. $2x^3 - 7x^2 + 5$ by $x - 3$

C. The **Remainder Theorem**

If the polynomial f(x) is divided by $x - c$, then the remainder, $\boldsymbol{r(x)}$, is the value $f(c)$.

Find the remainder of $f(x) = x^3 - 4x^2 - 5$ (you can check with division)

4. $x - 3$, c = ___
5. $x + 2$, c = ___

D. The **Factor Theorem**

Let f be a polynomial function. Then (x – c) is a factor of f(x) if and only if f(c) = 0. So, if f(c) = 0, then x – c is a factor of f(x) and c is a zero if f(x).

Use the factor theorem to determine if the function $f(x) = 2x^3 - x^2 + 2x - 3$ has the factor.

6. x – 1 7. x + 2

**II. Finding the Roots**

Consider P(x) = (x – 2)(x – 3)(x + 4), multiplying out the factors we get $P(x) = x^3 - x^2 - 14x + 24$. Where did the constant 24 come from?

So, the constants of the factors multiplied out give us the constant of P(x). If the product of the zeros equals the constant then the zeros are all factors of the constant! We use this fact in the form of the Rational Zeros Theorem.

A. **Rational Zeros Theorem**

If the polynomial, P, has integer coefficients, then every rational zero of P is of the form $\frac{p}{q}$ where p is a factor of the constant and q is a factor of the leading coefficient.

- find all possible ± p values and ± q values to make all the $\pm\frac{p}{q}$ ratios

  one or more of the ratios will be zeros of the polynomial

  determine zeros either by the remainder theorem $P(\frac{p}{q}) = 0$
- once you find a zero, use synthetic division to reduce the polynomials into factors
- keep following this process until you reach a quadratic factor then factor or use the quadratic formula to find the last two factors

1. Given $f(x) = 2x^3 + 11x^2 - 7x - 6$ , list the possible zeros, find the real zeros, then factor completely.

2. Solve $3x^3 + 8x^2 - 7x - 12 = 0$