2-3 Notes Polynomials Name ______________________

Pre-Calculus

I. Polynomials

A **polynomial** function of degree n is a function of the form:

$$\mathbf{P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}$$

where n is a nonnegative integer and

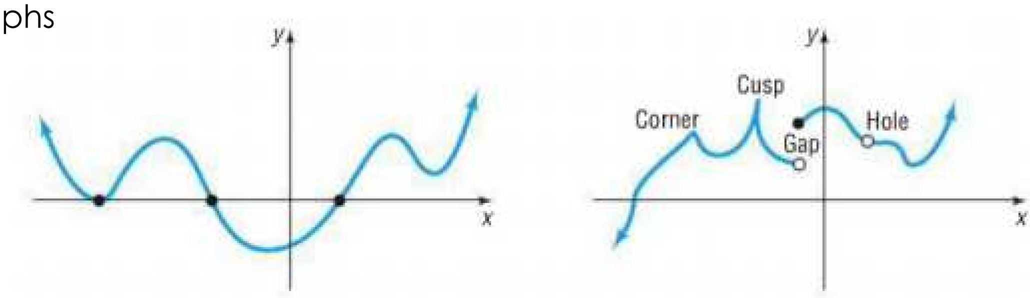
- the numbers $a_0, a_1, a_2, \ldots a_n$ are called coefficients of the polynomial
- the number a_0 is the constant term
- the number a_n, the coefficient of the highest power, is the **leading coefficient**
- the **degree** of the polynomial function is the largest power of x that appears

A. Identify the polynomial functions, state the degree.

$f(x) = 3x - 4x^3 + x^8$ $\qquad g(x) = \dfrac{x^2+3}{x-1}$ $\qquad h(x) = 5$

$F(x) = (x-3)(x+2)$ $\qquad G(x) = 3x - 4x^{-1}$ $\qquad H(x) = \dfrac{1}{2}x^3 - \dfrac{2}{3}x^2 + \dfrac{1}{4}x$

B. Graphs



(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function

II. Power Functions

A **power function** is a monomial function (a single term polynomial) $\mathbf{f(x) = ax^n}$, where n > 0. There are several basic polynomial functions we need to know.

$f(x) = 3x$ (degree 1) $\qquad f(x) = -5x^2$ (degree 2) $\qquad f(x) = 8x^3$ (degree 3) $\qquad f(x) = -5x^4$ (degree 4)

What is the significance of the leading coefficient?

A. Common power functions (parent functions)

$y = x$ $y = x^2$ $y = x^3$ $y = x^4$ $y = x^5$

** For even degree power functions, the graph is symmetric with respect to the y-axis so it is considered even. Odd degree power functions, the graph is symmetric with respect to the origin, so they are considered odd.**

B. Transformations of Monomials

State the transformations then sketch the graph. Determine the domain & range.

1. $f(x) = 1 - x^5$

2. $f(x) = \frac{1}{2}(x - 1)^4$

III. Zeros and Multiplicities

When we look for the zeros of a polynomial equation, we are looking for those values of x that are solutions to the equation or $P(x) = 0$. Graphically, we see the zeros where the graph touches or crosses the x-axis.

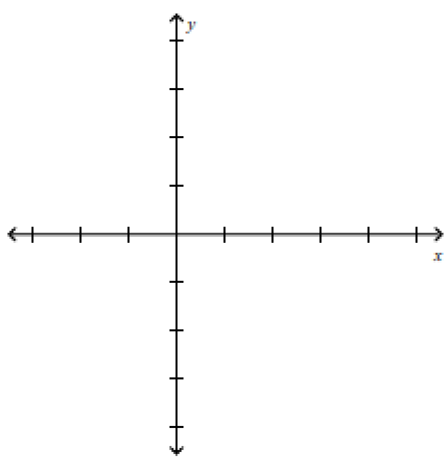
A. Real Zeros of Polynomials

If f is a polynomial and r is a real number for which $f(r) = 0$, then:

- r is a zero of f
- r is an x-intercept of the graph of f
- r is a solution of the equation $f(x) = 0$
- $(x - r)$ is a factor of $f(x)$

Use zeros to sketch the graph of a polynomial function of degree 3.

(zeros = - 3, 2, and 5)



What are the factors of this graph?

What is the polynomial function? (multiply factors)

Remember you can also take a polynomial function, factor it and then graph. To make this process easier, always remember to look for common factors of each term to factor out.

IV. Multiplicities

Given the factor (x – r): If (x – r) occurs more than once, r is called a repeated or multiple, zero of f. $(x - r)^m$ → or, r is a zero with a multiplicity of m.

Identify the zeros and their multiplicities. $P(x) = x^4(x - 2)^3(x + 1)^2$

A. Graphing

If the zero is a real number, then it will be an x-intercept.

- Multiplicity of a zero is EVEN → graph will TOUCH the x-axis at r
- Multiplicity of a zero is ODD → graph will CROSS the x-axis at r
- The higher the multiplicity the flatter the graph at the zero

Turning points:

- If $f(x)$ has a **degree of n**, then the graph of f has at most **n – 1** local extrema.
- If $f(x)$ has a **degree of n**, then the graph of f has at most **n – 1** turning points
- If a polynomial function f has **n – 1** turning points, the degree of f is at least n.

A polynomial of degree 5 will have <u>at most</u> 4 extrema or <u>at most</u> 4 turning points.

IT MAY NOT HAVE 4 TURNING POINTS OR EXTREMA!

Recall: Our local maximums and minimums; aka the extrema of a polynomial. These are the "hills" or "valleys" where the graph changes from increasing to decreasing or vice versa. An extrema is a y-value, not a point.

B. Which of the following graphs could be the graph of a polynomial function? For those that could, list the zeros, and state the least degree the polynomial can have.

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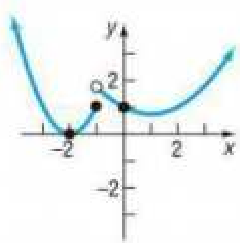

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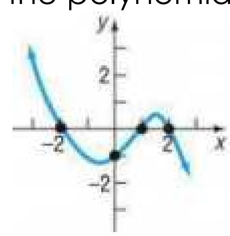

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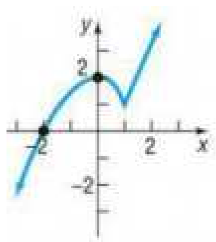

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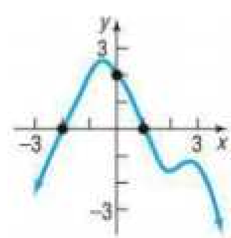

V. End Behavior

When we graph these polynomials, we put arrows on the end of the curve to show that the graph continues on to infinity. What is happening to the end of the graph? Is the graph rising (increasing) or falling (decreasing)? The end behavior of a polynomial is the description of what happens as x approaches infinity (the positive direction) and approaches negative infinity (the negative direction). We have a certain notation use to describe the end behavior.

$x \to \infty$ means x goes to infinity $x \to -\infty$ means x goes to negative infinity

For polynomials with degree ≥ 1 and odd If leading coefficient is **positive**, $a > 0$, the graph of f falls to the left and rises to the right as $x \to +\infty$ $f(x) \to +\infty$ as $x \to -\infty$ $f(x) \to -\infty$	**For polynomials with degree ≥ 2 and even** If $a > 0$ graph rises both to the left and right as $x \to \pm\infty$ $f(x) \to +\infty$
For polynomials with degree ≥ 1 and odd If leading coefficient is **negative**, $a < 0$, the graph of f rises to the left and falls t the right as $x \to +\infty$ $f(x) \to -\infty$ as $x \to -\infty$ $f(x) \to +\infty$	**For polynomials with degree ≥ 2 and even** If $a < 0$ graph falls both to the left and right as $x \to \pm\infty$ $f(x) \to -\infty$

VI. Analyze a Graph of a Polynomial Function

Analyze the graph of $f(x) = (2x + 1)(x - 3)^2$

1. Determine the end behavior.
2. Find the x- and y-intercepts
3. Determine the zeros of the function and their multiplicity.
4. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.
5. Determine the maximum number of turning points on the graph of the function.
6. Use the information in Steps 1-5 to draw a complete graph of the function by hand.