

## 4.5

## Linear Approximation Homework

Name \_\_\_\_\_  
Date \_\_\_\_\_ Period \_\_\_\_\_

**Problems 1 - 4, Find the tangent line approximation of each function at the given value  $x = a$ .**

1.  $f(x) = \tan x$ ; at  $a = \frac{\pi}{4}$

2.  $y = e^{\sqrt{x}}$ ; at  $a = 1$

3.  $g(x) = \frac{1}{\sqrt{1+x}}$ ; at  $a = 3$

4.  $y = \tan^{-1} x$ ; at  $a = 1$

5. Let  $f$  be a differentiable function such that  $f(-2) = 3$  and  $f'(-2) = 6$ . The tangent line to the graph of  $f$  at  $x = -2$  can be used to find an approximation to a zero of the function,  $f$ . Find the approximation for the value of  $x$  such that  $f(x) = 0$ .

6. Find the linear approximation for  $f(\theta) = \sin^2 \theta$  for  $\theta = \frac{\pi}{4}$ . Use the linear approximation to estimate  $\theta = 0.8$  and calculate the error. Determine whether the approximation overestimates or underestimates the function.

7. Find the linearization of the function  $f(x) = \sqrt{x+3}$  at  $x = 1$  and use it to approximate  $\sqrt{4.05}$ . Determine whether the approximation is an overestimate or underestimate. Explain.

8. Let  $f$  be a function that is twice differentiable for all real numbers. The table below gives values of  $f$  for several points in the interval  $1 \leq x \leq 10$ .

$x$	1	3	6	9	10
$f(x)$	2	4	-3	1	3

A. Estimate  $f'(5)$ . Show work that leads to your answer.

B. Suppose  $f'(6) = 2$  and the function  $f$  is concave up on the closed interval  $6 \leq x \leq 9$ . Use the tangent line at  $x = 6$  as an approximation to show that  $f(8) \leq 1$ .

C. Write the secant line for the graph of  $f$  on  $6 \leq x \leq 9$  to show that  $f(8) \geq -\frac{1}{3}$ .

9. Use the table, at right, to complete the following.  
Given  $P(x) = f(g(x))$

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	-2	1	4
-1	1	-1	-2	1
0	-1	2	-1	-2

- A. Write the equation of the tangent line at  $x = -1$ .
- B. Use the method of linear approximation to estimate the value of  $P(-1.5)$ .
- C. If  $P(-2) = 5$ , what does this value tell you about the behavior of the graph of  $P(x)$  when  $x = -1.5$ ? Explain how you know.

10. Use linearization to estimate the given numbers. Find the error of the approximation.

- A.  $\ln(1.06)$
- B.  $(8.05)^{2/3}$

11. The local linear approximation to the function  $g$  at  $x = -3$  is  $y = 4x - 7$ .  
What is the value of  $g(-3) + g'(-3)$ ?

12. Find a linear approximation for  $f(x) = 2x e^{2x-6}$  at  $x = 3$ . Then, use your equation to estimate  $f(3.1)$ . Find the error.

13. Let  $g(x) = x^2 - x - 12$ . The tangent line to the graph of  $g$  at  $x = 2$  is used to approximate values of  $g(x)$ . Which of the following is the greatest value of  $x$  for which the error from this tangent line approximation is less than 0.5?
- A. 1.3      B. 2.6      C. 2.7      D. 2.8

14. Given the implicit curve,  $x^2 + y^3 = 2x^2y$ . At the point  $(1, 1)$  write the equation for the tangent line and use it to approximate  $y(1.2)$ .

15. Given  $f'(x) = \frac{6x^4}{x^2+1}$  and  $f(1) = 8$ . Use a linear approximation to estimate  $f(1.02)$