Lesson 4: Related Rates

Topic 4.5: Solving Related Rates Problems

The derivative, $\frac{dy}{dx}$, of a function, $y = f(x)$, is its instantaneous rate of change with respect to the variable x.

- ☐ When a function describes either position or distance, its rate of change is interpreted as velocity.
- ☐ In general, a time rate of change answers the question: *How fast is a quantity changing?*
  - For example, if V is volume that is changing in time then $\frac{dV}{dt}$ is the rate, or how fast, the volume is changing with respect to time.
  - If a person is walking toward a street-lamp at a constant rate of 3 feet per second, then we know that the distance is decreasing, so $\frac{dx}{dt} = -3\frac{ft}{sec}$.
  - If they walk away from the lamp then the distance is increasing and the rate of change becomes positive or $\frac{dx}{dt} = 3\frac{ft}{sec}$.

GUIDELINES FOR SOLVING RELATED RATE PROBLEMS:

1. **Make a sketch and label the quantities.**
2. **Read the problem and identify all quantities as: "KNOW", "GIVEN", and "FIND" with the appropriate information.**
3. **Write an equation involving the variables whose rates of change either are given or are to be determined.**
4. **Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time, t*.**
5. ***AFTER COMPLETING STEP 4*, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.**

EX #1: Suppose x and y are both differentiable functions of t and are related by the equation $y = x^2 - 3x$. Find $\frac{dy}{dt}$ when $x = 3$ given that $\frac{dx}{dt} = 2$, when, $x = 3$.

EX. #2: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

1. KNOW:

2. GIVEN:

3. FIND:

EX. #3: Air is being pumped into a spherical balloon at a rate of 800 cubic centimeters per minute. How fast is the radius of the balloon changing at the instant the radius is 30 centimeters?

1. KNOW:

2. GIVEN:

3. FIND:

EX. #4: The top of a 25- foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

1. KNOW:
2. GIVEN:
3. FIND:

EX. #5: At noon, ship A is 150 km east of ship B. Ship A is sailing west at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4 p.m.?

1. KNOW:
2. GIVEN:
3. FIND:

EX #6: A right circular cylinder is changing shape. The radius is decreasing at a rate of 2 inches/sec while its height is increasing at the rate of 5 inches/ sec. When the radius is 4 inches and the height is 6 inches, how fast is the volume changing?

Given: $V = \pi r^2 h$

1. KNOW:
2. GIVEN:
3. FIND:

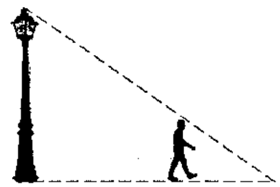
EX #7: An inverted cone is leaking water at a rate of 1 cm^3/min. The cone has a height of 9 cm and a diameter of 6 cm. Find the rate at which the water level is dropping when $h = 3\ cm$.

Given: $V = \frac{1}{3}\pi r^2 h$

1. KNOW:
2. GIVEN:
3. FIND:

EX #8: A lamppost is 26-feet above the sidewalk. Andy Grate is a 6-foot tall Calculus teacher and is walking away from the lamp at a rate of 4 feet per second. When he is 20 feet away from the lamppost, find each of the following.

A. How fast is the length of his shadow changing?



B. At what rate is the tip of his shadow moving?